

Diversity of Vocabulary and Homogeneity of Style in *Tirant lo Blanc*

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Abstract

Tirant lo Blanc is the main work in catalan literature, and it is considered by some to be the first modern novel in Europe. To help settle the debate around its authorship, we analyze the evolution of the diversity of various measures of the vocabulary used, evaluated first on consecutive blocks of text of 1000 words and second on the chapters of the book. The analysis reveals a sharp boundary, that is estimated through change-point techniques to be around chapter 382, and might indicate a switch in authorship. The language used after that point is a lot less rich and diverse than before.

Keywords: stylometry, diversity indexes, Simpson index, change point estimation, switch regression model.

1. Introduction

Tirant lo Blanc is a chivalry book hailed to be “the best book of its kind in the world” by Cervantes in *Don Quixote*, and considered by some to be the first modern novel in Europe, (see Vargas Llosa, 1991 and 1993). It is the main work in catalan literature, and its main body was written between 1460 and 1464, but it was not printed until 1490. There has been an intense and long lasting debate around its authorship originating from conflicting information given in its first edition. Where in its dedicatory letter it is stated that the whole book is the work of Martorell (1413-1468), in the colophon of that first edition it is stated that the last one fourth of the book was written by Martí Joan de Galba, after the death of Martorell.

The degree of participation of Galba in the book has been the object of a long lasting and intense debate. Articles like Martínez Martínez (1916), Entwistle (1927), Moll (1933), Menéndez Pelayo (1943), Riquer (1947), Coromines (1956), Nicolau d'Olwer (1961), Goertz (1967), Bosch (1987), Ferrando (1989 and 1995), Rubiera (1990 and 1992) and Wittlin (1990 and 1993) favor a literal interpretation of the “colophon” defending the hypothesis of a change of author between chapters 300 and 350, and thus that Galba is the author of the last one fifth of the book, composed by 487 chapters. On the other hand, articles like Givanel (1916), Vaeth (1918), Marinesco (1979), Riquer (1990), Hauf (1993), Chiner (1991 and 1993), Badia (1993), Casanova (1994) favor the hypothesis that the whole book was written by a single author, that is, by Martorell. See also Guia (1995 and 1996) for an heterodox interpretation of this question. For a nice overview of the debate around this authorship problem, see Riquer (1990) and Chiner (1993).

Neither of the two candidate authors left any text comparable to the one under study, and therefore one can not use discriminant analysis to help classify the chapters in the book by author. By analyzing the style in sample texts encompassing about ten percent of the book, Ginebra and Cabos (1998) detected heterogeneities that might indicate the existence of two authors. Through

the analysis of the multinomial sequences associated to the distribution of word length and to the use of the most frequent words, Riba (2002), Riba and Ginebra (2003) and Giron, Ginebra and Riba (2004) find a sharp stylistic boundary somewhere between chapters 371 and 382.

In this paper we characterize the style of *Tirant lo Blanc* through the diversity of its vocabulary, measured from the list of all the words that appear in the book, and the number of occurrences of each word. Section 2 presents measures of the diversity of the vocabulary and Section 3 explores the evolution of these measures evaluated in consecutive blocks of text of 1000 words, as well as on the chapters that have very unequal lengths. That reveals a sharp boundary, that is estimated through change-point techniques to be around chapter 382.

2. The Measure of the Diversity of the Vocabulary

To characterize the style of an author through the diversity of its vocabulary, the basic assumption is that he has available a certain stock of words that he uses in his writings, some of which he favors more than others. For each word i , let π_i be the proportion of times that it would be found in a text of infinite length of that author. It is assumed that both the list of words that form the author's hypothetical vocabulary as well as the values of π_i are constant along all the writings under study of an author. This hypothesis will rarely hold whenever these writings embrace different epochs and/or genres.

If we sample texts produced by a writer, his vocabulary will be reflected in the sample frequency of each word that appears in the texts. Let N denote the total number of words in a given text, let n_i denote the number of occurrences of word i , and let $\hat{\pi}_i = n_i/N$ denote the proportion of occurrences of i in that sample text. Obviously N is equal to the sum of n_i over all words i in his vocabulary. If the hypothesis of stationarity applies, the expected value of $\hat{\pi}_i$ is π_i , and therefore it does not depend on N .

Furthermore, given a sample text of length N , let V denote the number of different words and let V_r denote the number of different words appearing exactly r times in the text. Note that the sum of V_r over all integers r is equal to V and that the sum of rV_r over them is equal to N . The proportion of different words appearing r times in a sample text (of length N) is $\hat{p}_r = V_r/V$, and its expected value is denoted by $p_r(N)$. Note that the distribution of \hat{p}_r , and in particular its expected value, depends on N . Modelling the vocabulary through \hat{p}_r is more convenient than doing it through $\hat{\pi}_i$, because the support of the random variable $\hat{\pi}_i$ encompasses both the observed vocabulary as well as the unobserved one, and therefore it is unknown, while the support of \hat{p}_r is a subset of the integers.

Given a text with a total of N words, the larger V , the most rich and diverse its vocabulary, and a very simple measure of diversity is V itself. Observe though that the distribution of V depends on N , and in particular note that while N could increase without limit, V is limited by the number words in the vocabulary of the author. Thus, V by itself can only be used to compare texts of the same length.

Given N and V fixed, the larger p_r for small r and the smaller p_r for large r , more diverse its vocabulary. Any diversity measure should respect these two ordering levels. Two alternative diversity measures are V_1 , that is the number of words appearing once, and V_2 , the number of words appearing twice. Given N , the larger V_1 and V_2 , the more diverse the vocabulary, but since their distributions also depend heavily on N (see Sichel, 1986a), they are not convenient when comparing texts of different lengths.

An alternative measure is the entropy of the the vocabulary of a sample text, defined as:

$$H = - \sum_i \hat{\pi}_i \log \hat{\pi}_i = - \sum_i \frac{n_i}{N} \log \frac{n_i}{N} = - \sum_r \frac{rV_r}{N} \log \frac{r}{N}.$$

The larger H , the more diverse the vocabulary, but for a given author, as N increases $E(H)$ decreases and is different from the entropy of the vocabulary of the author. That means that H is not useful either, when comparing texts of different lengths.

Instead, Simpson (1949) proposes measuring diversity through the probability that picking up two words at random from a sample text, they are the same word, D , that can be computed as:

$$D = \frac{\sum_r r(r-1)V_r}{N(N-1)} = \frac{\sum_i n_i(n_i-1)}{N(N-1)}.$$

This index is also called Gini's Index, because it was considered in another context by Gini (1912). The more diverse the vocabulary, the smaller the probability of repeating words, and therefore the smaller D tends to be. What makes the Simpson index, D , more useful than V , V_1 or V_2 is that its expected value depends on the diversity of the vocabulary of the author, but it does not depend on N , and that makes it a very nice tool when comparing texts of different lengths. In fact, the expected value of D is the probability that two words picked up at random from a text of infinite length are the same word,

$$E(D) = \sum_i \pi_i^2.$$

Sichel (1986a) recommends evaluating all these diversity measures based only on words of a special kind, like nouns, prepositions or conjunctions, but we do it based on all words instead. Following the example of Mosteller and Wallace (1984), we do not lemmatize *Tirant lo Blanc* and thus we do not reduce all verbs to their infinitive, all nouns to their singular and all adjectives to the same genre. For more details on the measure of diversity, see Good (1953), Good and Toulmin (1956), Margalef (1958), Efron and Thistedt (1979 and 1987), Patil and Taillie (1982), Pielou (1982), Harris (1982), Rao (1982), Holmes (1985) and Sichel (1986a and 1986b), Manly (1994) and Lebart *et al.* (1998), Alvarez, Bécue and Lanero (2000), Hubert *et al.* (2002).

3. Diversity of the Vocabulary in *Tirant lo Blanc*

Here we use the modern edition of *Tirant lo Blanc* by Riquer (1983), because the original manuscript is not available in digital format. Given the length of this manuscript and the unavailability of a printed reproduction of the first edition of the book, it was beyond our means introduce that edition available in digital format. Even though it would be clearly preferable to do the analysis based on the edition of 1490, we believe that finding a stylistic boundary on a modern edition where some experts believe there is a change of author, is very much worth reporting.

After excluding the titles of the chapters and words in italics, that correspond to quotations in latin, the book amounts to a total of 396349 words. In that edition, the book is broken into 487 chapters of very unequal lengths, with 19 chapters exclusively in italics and a longest chapter of 6521 words.

3.1. Change-point Analysis through Blocks of Text of Constant Length

To avoid the effect of the dependency of the distribution of the diversity measures on text length, we divide *Tirant* in 396 blocks of $N = 1000$ consecutive words, discarding the last 349 words of the book from consideration. Some blocks include more than one chapter while some chapters are split in several blocks.

For each of the blocks, we compute the value of V , V_1 , V_2 , H , and D . Clearly, the larger V and H and the smaller D , the more diverse and rich the vocabulary in the corresponding block. Figure 1 presents the relation between these 396 values of V , V_1 , V_2 , H , and D . Given that V_2 is not correlated with the other diversity measures, we will not use it in the rest of the analysis.

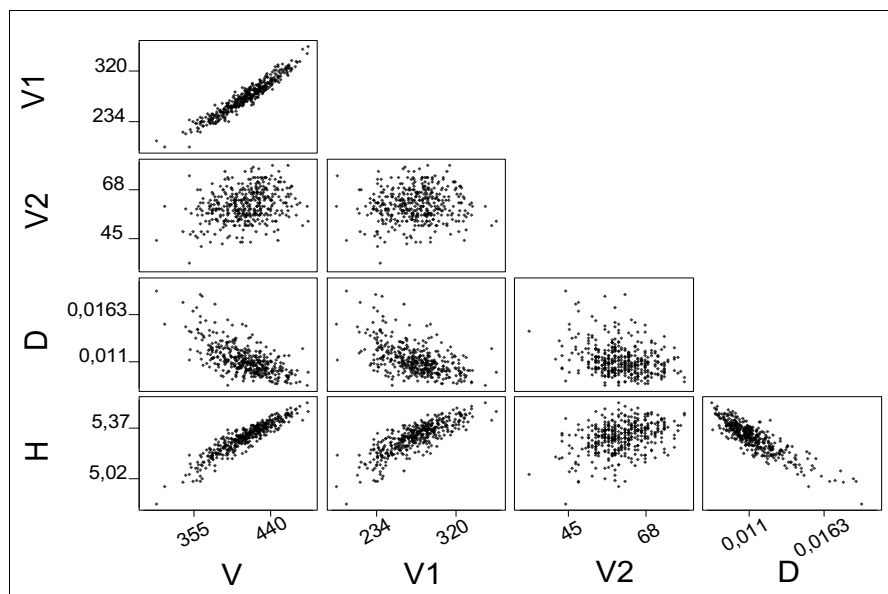


Figure 1. Relation between the values of V , V_1 , V_2 , H , and D for the 396 blocks of 1000 consecutive words that form *Tirant lo Blanc*.

Figure 1 presents the sequence of values taken by V , H and D in these blocks of constant length. The three indices present a clear shift in their level at around the same point. In the first part of the book, V and H tend to be larger and D tends to be smaller than in the last part of it, and thus the vocabulary of the main part of the book is richer and more diverse than the vocabulary used at the end of the book. Next, the change-point in these sequences of diversity measures is estimated using the maximum likelihood criteria.

Let y_1, y_2, \dots, y_n be an ordered sequence of mutually independent random variables, with distribution function $F_{\theta_0}(y)$ for $i = 1, \dots, r$, and distribution function $F_{\theta_1}(y)$ for $i = r + 1, \dots, n$, where θ_0 , θ_1 and r are unknown. The estimation of change-point r has been extensively studied for various univariate distributions and very different points of view. For a likelihood based approach, see for example Hinkley (1970 and 71), for a non-parametric approach see Wolfe and Schechtman (1984) while for presentations of the Bayesian approach see Smith (1975 and 80). For a review, see Pettit (1989).

In our case, y_i would be the values V_i , H_i and D_i taken by V , H and D in each block. If all the chapters were written by the same author, it would be reasonable to expect the distributions for V_i , H_i and D_i to be the same for all i . On the other hand, if there was a change of author in block $i = r$, one might detect a change in distribution at that point. We assume that

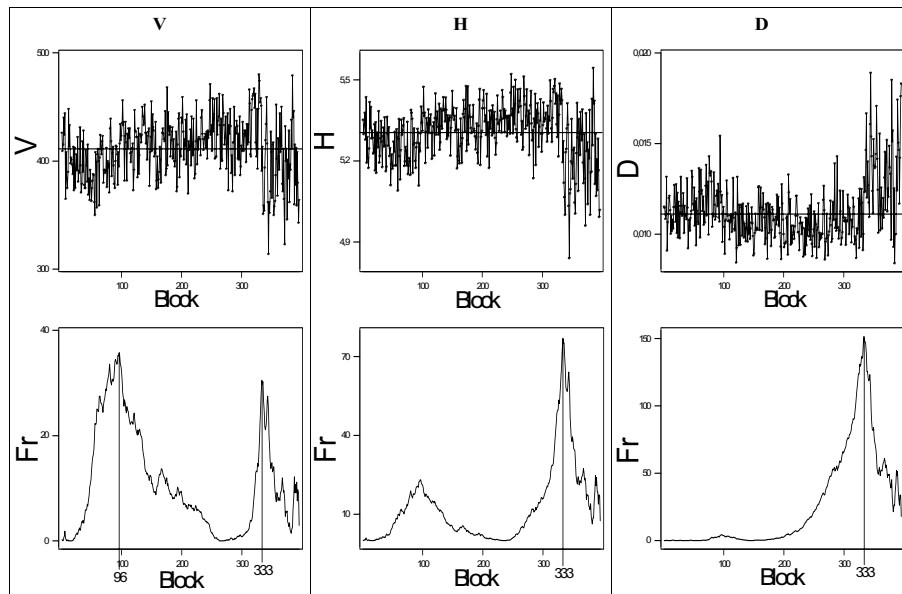


Figure 2. Sequence of the values taken by V , H and D , for the blocks of 1000 consecutive words. The corresponding values of the F statistic for the single-switch normal regression models, (1), as a function of the switch-point, r , are presented below. Block 333 corresponds to chapter 382.

these observations are conditionally independent and normally distributed, and that the change in distribution translates into a sudden shift in its expected value. In order to estimate r , we fit all the single-switch normal regression models:

$$V_i \sim N(\beta_0^r + \beta_1^r I r_i, \sigma^2), \quad (1)$$

with $I r_i = 0$ for $i \leq r$ and $I r_i = 1$ for $i > r$, that assume $E(V_i) = \beta_0^r$ for $i \leq r$ and $E(V_i) = \beta_0^r + \beta_1^r$ for $i > r$. The change-point r is then estimated to be the one under which (1) fits the observed sequence V_i . As the goodness of fit criteria we use the F -statistic from the ANOVA table, F_r ; and the value of r that maximizes F_r is the maximum likelihood estimate of the change-point of a sequence of mutually independent normal observations. We also repeat this exercise for the sequences of H_i and of D_i .

Figure 2 presents the values of F_r as a function of r for these sequences. One can observe how for the three sequences there is a clear maximum for the block numbered $i = 333$, that corresponds to chapter 382. Given that our diversity indices are all positive, we repeated this analysis using single-switch gamma regression models instead of normal ones, and we obtained very similar results to the ones reported here. For a cluster analysis of these data, that reinforces the evidence in favor of the existence of this stylistic boundary, see Riba (2002).

3.2. Change-point Analysis of the Diversity Measured in Chapters

When one uses the chapters of the book as the unit of interest instead of blocks of constant length, one needs to take the length of each chapter i , N_i , into consideration.

Figure 3 presents the relation between D_i and N_i and the time plot of D_i for the 425 chapters of more than 200 words. Given that $E(D_i)$ does not depend on N_i , one can estimate the change-point in that sequence using an approach analogous to the one in Section 3. That is, one fits the single-switch linear normal model (1) for each $r = 1, \dots, 424$, through weighted least squares, (because the variance of D_i is proportional to $1/N_i$), and one estimates the change-point to be

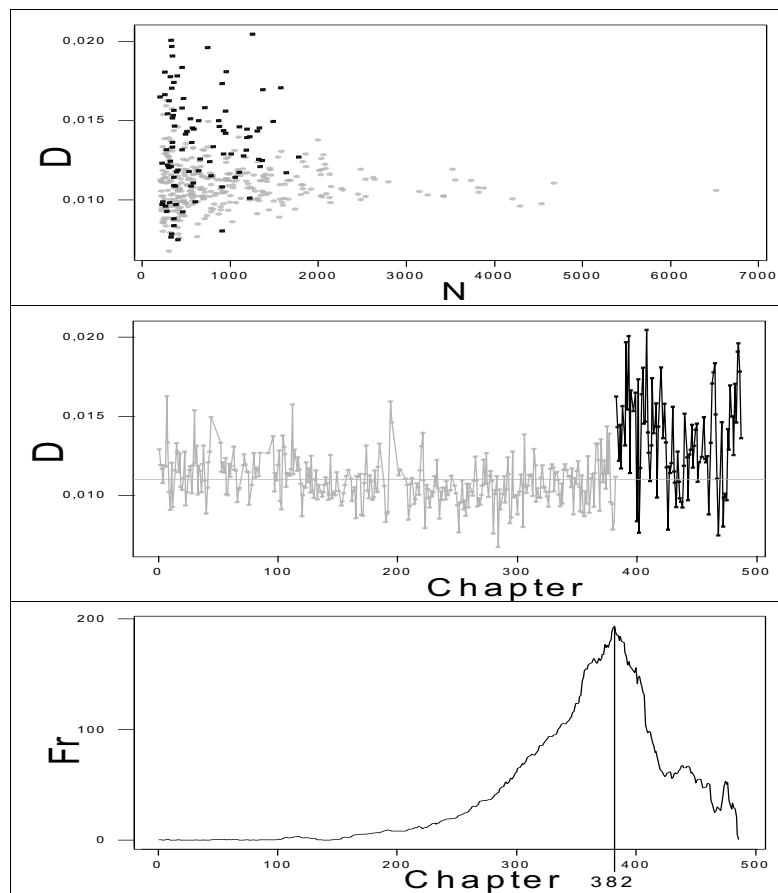


Figure 3. The top panel presents the relation between the Simpson index for chapter i , D_i , and the total number of words in it, N_i . The middle panel presents the sequence of D_i . The bottom panel presents the F statistic for (1) as a function of r for this sequence. Gray indicates chapters 1 to 382 while black indicates chapters 383 to 487.

the one that corresponds to the model with the largest F . It turns that the best fit for the observed sequence is again obtained for $r = 382$.

Figure 4 relates the values taken by V and H in chapter i , V_i and H_i , with N_i , thus illustrating the vocabulary growth when the text length grows. From that figure, it is clear that in the logarithmic scales there is a strong linear relationship between V and N and between H and N , even though the vocabulary growth seems to be different for the chapters at the end of the book, in black, than for the ones at the beginning of it, in gray. That relationship is even more linear when using $H_s = H/\log N$ against $\log N$ instead of $\log H$ against $\log N$ (see Holmes, 1985).

Here we use the relation between V_i and N_i and between H_i and N_i to characterize the vocabulary of an author, through their vocabulary growth. By searching for the chapter where these relations most likely change, we find new estimates of the stylistic boundary. More specifically, we fit the single-switch linear normal regression model:

$$\log V_i \sim N(\beta_0^r + \beta_1^r \log N_i + \beta_2^r I_{r_i} + \beta_3^r I_{r_i} * \log N_i, \sigma^2), \quad (2)$$

for all $r = 1, \dots, 424$, that assume that $E(\log V_i) = \beta_0^r + \beta_1^r \log N_i$ for $i \leq r$ and $E(\log V_i) = \beta_0^r + \beta_2^r + (\beta_1^r + \beta_3^r) \log N_i$ otherwise. Again, r is estimated to be the change-point for which the corresponding model fits the observed V_i best. We repeat this same estimation procedure for H_i . In both instances, the change-point is estimated to be again $r = 382$.

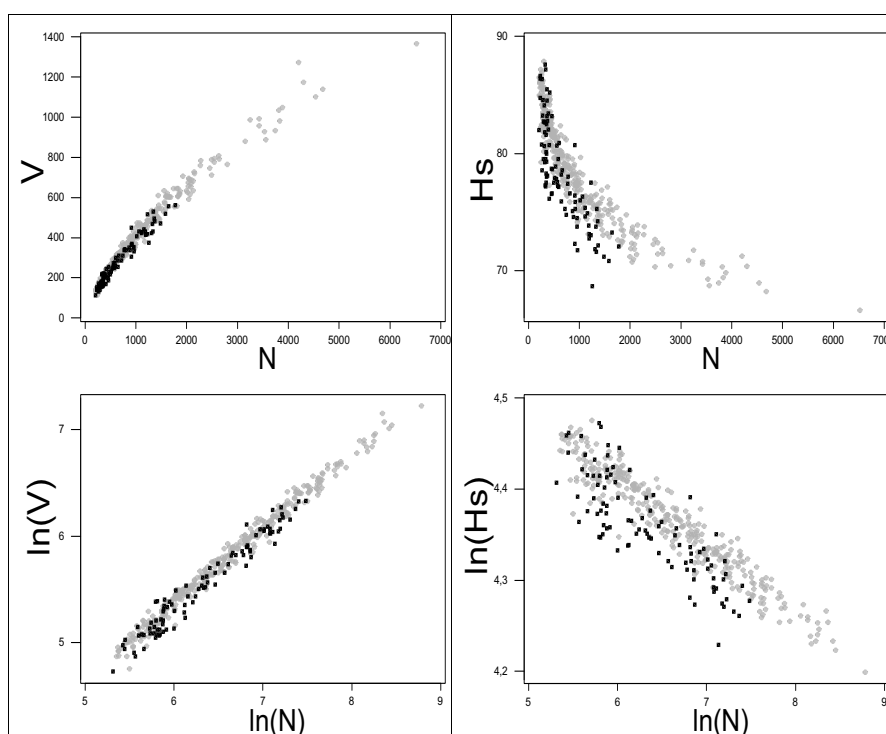


Figure 4. Relation between the number of different words in chapter i , V_i , and N_i , and of H_i and N_i , for the 425 chapters of more than 200 words, illustrating the vocabulary growth in *Tirant lo Blanc*. Gray dots correspond to the chapters before chapter 382, while the black dots identify the ones after it.

4. Conclusions

Despite the use of a modern edition of *Tirant lo Blanc*, we detect a very clear stylistic boundary near chapter 382, in very close agreement with the result of the statistical analysis on other variables, reported in Riba (2002), Riba and Ginebra (2003) and Giron, Ginebra and Riba (2004). That boundary is placed very close to where is anticipated by the colophon of the first edition of the book, and where it was guessed by some experts on the book. It is important to remark that even though the statistical analysis supports the existence of two authors, it is not up to us to exclude the possibility that the stylistic boundary found could be explained otherwise. Also, the fact that a few chapters at the end of the book have diversity indices that are similar to the ones in the chapters at the beginning of it, might mean that the second author finished the book by filling in material at the end of it.

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